

Trigonometric Identities & Equations

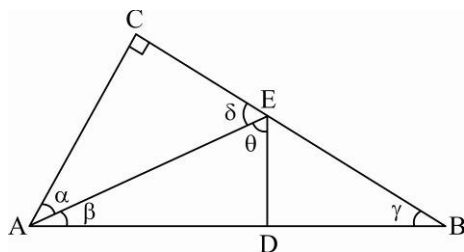
Miscellaneous Question Bank

Level – 1 & 2

- The function f given by $f(x) = \sin\left(\frac{\pi x}{2}\right) + 2\cos\left(\frac{\pi x}{3}\right) - \tan\left(\frac{\pi x}{3}\right)$ is periodic with period :
 (A) 6 (B) 3 (C) 4 (D) 12
- In a right-angled triangle, the hypotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex on the hypotenuse, then the other two angles are :
 (A) $\frac{\pi}{4}, \frac{\pi}{4}$ (B) $\frac{\pi}{8}, \frac{3\pi}{8}$ (C) $\frac{\pi}{3}, \frac{\pi}{6}$ (D) None of these

PARAGRAPH FOR QUESTIONS 3 - 5

In the figure below, it is given that $\angle C = 90^\circ$, $AD = DB$, ED is perpendicular to AB , $AB = 20$ units and $AC = 12$ units.



- Area of triangle AEC is :
 (A) 24 sq. units (B) 21 sq. units (C) 42 sq. units (D) $\frac{21}{2}$ sq. units
- The value of $\tan(\delta + \beta)$, is :
 (A) $-\frac{117}{44}$ (B) $\frac{17}{4}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$
- The value of $\cos(\alpha + \beta)$ is :
 (A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{117}{125}$ (D) $-\frac{44}{125}$
- If $P = \cos\frac{\pi}{20} \cdot \cos\frac{3\pi}{20} \cdot \cos\frac{7\pi}{20} \cdot \cos\frac{9\pi}{20}$ and $Q = \cos\frac{\pi}{11} \cdot \cos\frac{2\pi}{11} \cdot \cos\frac{4\pi}{11} \cdot \cos\frac{8\pi}{11} \cdot \cos\frac{16\pi}{11}$, then $\frac{P}{Q}$ is :
 (A) Not defined (B) 1 (C) 2 (D) None of these
- A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B, C are in A.P. then the angles A, B, C are respectively:
 (A) $\frac{5\pi}{12}, \frac{\pi}{4}, \frac{\pi}{3}$ (B) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$ (C) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (D) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{4}$

8. If $\sin \theta + \cos \theta = \frac{1}{5}$ and $0 < \theta < \pi$, then $\tan \theta$ is :
- (A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$
- *9. If $\sin \theta + \sin \phi = a$ and $\cos \theta + \cos \phi = b$, then :
- (A) $\cos\left(\frac{\theta - \phi}{2}\right) = \pm \frac{1}{2} \sqrt{a^2 + b^2}$ (B) $\cos\left(\frac{\theta - \phi}{2}\right) = \pm \sqrt{a^2 - b^2}$
- (C) $\cos\left(\frac{\theta - \phi}{2}\right) = \pm \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$ (D) $\cos(\theta - \phi) = \frac{a^2 + b^2 - 2}{2}$
- *10. If $\sin(x - y) = \cos(x + y) = \frac{1}{2}$ then the values of x and y lying between 0 and π are given by :
- (A) $x = \frac{\pi}{4}, y = \frac{3\pi}{4}$ (B) $x = \frac{\pi}{4}, y = \frac{\pi}{12}$ (C) $x = \frac{5\pi}{4}, y = \frac{5\pi}{12}$ (D) $x = \frac{11\pi}{12}, y = \frac{3\pi}{4}$
11. If $S_n = \cos^n \theta + \sin^n \theta$, then the value of $3S_4 - 2S_6$ is given by :
- (A) 4 (B) 0 (C) 1 (D) 7
12. If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta = y \cos \theta$ then $x^2 + y^2$ is :
- (A) 2 (B) 3 (C) 4 (D) 1
13. If $\cos x + \cos^2 x = 1$, then the value of $\sin^{12} x + 3 \sin^{10} x + 3 \sin^8 x + \sin^6 x - 1$, is equal to :
- (A) 2 (B) 1 (C) -1 (D) 0
14. The minimum value of $f(x) = \sin^4 x + \cos^4 x$, $0 \leq x \leq \frac{\pi}{2}$ is :
- (A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{4}$ (C) $\frac{-1}{2}$ (D) $\frac{1}{2}$
15. The solution of the equation $4 \sin^4 x + \cos^4 x = 1$ is :
- (A) $x = 2n\pi$ (B) $x = n\pi + \frac{\pi}{2}$ (C) $x = n\pi$ (D) None of these
16. If A, B, C, D are the angles of a cyclic quadrilateral, then $\cos A + \cos B + \cos C + \cos D =$
- (A) 4 (B) 1 (C) 0 (D) -1
17. If $P_n = \cos^n x + \sin^n x$, then $2P_6 - 3P_4 + 1 =$
- (A) 2 (B) 3 (C) 0 (D) 1
18. If $\tan \theta_1 = k \cot \theta_2$ then $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)}$ is equal to:
- (A) $\frac{1+k}{1-k}$ (B) $\frac{1-k}{1+k}$ (C) $\frac{k+1}{k-1}$ (D) $\frac{k-1}{k+1}$
19. Range of $3 \sin 2x + 4 \cos 2x + 3$ is:
- (A) [2, 8] (B) [-5, 8] (C) [0, 8] (D) [-2, 8]

20. $\frac{\cos A + \cos 3A + \cos 5A + \cos 7A}{\sin A + \sin 3A + \sin 5A + \sin 7A}$ is equal to:
 (A) $\sin 4A$ (B) $\cos 4A$ (C) $\tan 4A$ (D) $\cot 4A$
21. Minimum value of $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$ is :
 (A) 0 (B) 2 (C) 7 (D) 9
22. The solutions of the system of equations $x + y = \frac{2\pi}{3}$ and $\cos x + \cos y = \frac{3}{2}$ where x and y are real, are :
 (A) $x = \pi, y = -\frac{\pi}{3}$ (B) $x = -\frac{\pi}{3}, y = \pi$ (C) No solution (D) None of these
23. If $\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2$ then x is equal to:
 (A) π (B) $\pi/3$ (C) $\pi/4$ (D) $\pi/6$
24. One root of the equation $\cos x + x = 0$ lies in the interval :
 (A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, 0\right)$ (C) $\left(\frac{\pi}{2}, \pi\right)$ (D) $\left(\pi, \frac{3\pi}{2}\right)$
25. If $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$ for $0 \leq \theta \leq \pi$ then θ is equal to:
 (A) $\frac{\pi}{7}, \frac{5\pi}{7}, \pi$ (B) $\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}$
 (C) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{\pi}{3}, \frac{2\pi}{3}$ (D) $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$
- *26. If $\tan \theta = x - \frac{1}{4x}$, then $\sec \theta - \tan \theta$ is equal to:
 (A) $-2x$ (B) $-\frac{1}{2x}$ (C) $2x$ (D) $\frac{1}{2x}$
27. If $D = \begin{vmatrix} 1 & \cos \theta & 1 \\ -\sin \theta & 1 & -\cos \theta \\ -1 & \sin \theta & 1 \end{vmatrix}$ then D lies in the interval :
 (A) $[0, 4]$ (B) $[2, 4]$ (C) $[2 - \sqrt{2}, 2 + \sqrt{2}]$ (D) $[-2, 2]$
28. $\tan^6 \frac{\pi}{9} - 33 \tan^4 \frac{\pi}{9} + 27 \tan^2 \frac{\pi}{9}$ is equal to:
 (A) $\tan \frac{\pi}{3}$ (B) $\tan^2 \frac{\pi}{3}$ (C) $\tan \frac{\pi}{6}$ (D) $\tan^2 \frac{\pi}{6}$
29. If $\sin \alpha + \sin \beta = a$, $\cos \alpha + \cos \beta = b$ then $\sin(\alpha + \beta)$ is equal to:
 (A) ab (B) $a + b$ (C) $\frac{2ab}{a^2 - b^2}$ (D) $\frac{2ab}{a^2 + b^2}$
30. $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15}$ is equal to:
 (A) $1/4$ (B) $1/8$ (C) $1/16$ (D) $1/32$

- 31.** A quadratic equation whose roots are $\sin^2 18^\circ, \cos^2 36^\circ$ is :
- (A) $16x^2 - 12x + 1 = 0$ (B) $x^2 - 12x + 1 = 0$
(C) $16x^2 - 12x - 1 = 0$ (D) None of these
- 32.** The general solution of $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \cdot \tan 4\theta \cdot \tan 7\theta$.
- (A) $\theta = \frac{n\pi}{4}$ (B) $\theta = \frac{n\pi}{12}$ (C) $\theta = \frac{n\pi}{6}$ (D) none of these
- 33.** The smallest positive $x (y > 0)$ satisfying $x - y = \frac{\pi}{4}, \cot x + \cot y = 2$ is $\frac{5\pi}{\lambda}$, the numerical integer λ is:
- (A) 15 (B) 14 (C) 12 (D) 13
- 34.** If $\cos \frac{\pi}{7}, \cos \frac{3\pi}{7}, \cos \frac{5\pi}{7}$ are the roots of the equation $8x^3 - 4x^2 - 4x + 1 = 0$. The equation whose roots are $\cos \frac{2\pi}{7}, \cos \frac{4\pi}{7}, \cos \frac{6\pi}{7}$ is:
- (A) $8x^3 + 4x^2 + 4x - 1 = 0$ (B) $8x^3 + 4x^2 + 4x + 1 = 0$
(C) $8x^3 + 4x^2 - 4x - 1 = 0$ (D) $8x^3 - 4x^2 + 4x - 1 = 0$
- 35.** If $x + y = 3 - \cos 4\theta$ and $x - y = 4 \sin 2\theta$ then:
- (A) $x^4 + y^4 = 9$ (B) $\sqrt{x} + \sqrt{y} = 16$
(C) $x^3 + y^3 = 2(x^2 + y^2)$ (D) $\sqrt{x} + \sqrt{y} = 2$
- 36.** Given $a^2 + 2a + \operatorname{cosec}^2 \left(\frac{\pi}{2} (a + x) \right) = 0$ then, which of the following holds good?
- (A) $a = 1; \frac{x}{2} \in I$ (B) $a = -1; \frac{x}{2} \in I$
(C) $a \in R; x \in \phi$ (D) a, x are finite but not possible to find
- 37.** If $A = \sin \frac{2\pi}{7} + \sin \frac{4\pi}{7} + \sin \frac{8\pi}{7}$ and $B = \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{8\pi}{7}$ then $\sqrt{A^2 + B^2}$ is equal to:
- (A) 1 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{3}$
- 38.** In a triangle ABC, angle $B < \text{angle } C$ and the values of B and C satisfy the equation $2 \tan x - k(1 + \tan^2 x) = 0$ where $(0 < k < 1)$. Then the measure of angle A is:
- (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$
- 39.** If $a \sin x + b \cos x = 1$ and $a^2 + b^2 = 1$ ($a, b > 0$), then consider the following statements:
I. $\sin x = a$ II. $\tan x = a/b$ III. $\tan x = b$
(A) only III is false (B) only I is true
(C) All of I, II, III must be true (D) None of I, II or III is correct.
- 40.** If $\frac{\cos 3x}{\cos x} = \frac{1}{3}$ for some angle $x, 0 \leq x \leq \frac{\pi}{2}$, then the value of $\frac{\sin 3x}{\sin x}$ for some x , is:
- (A) $\frac{7}{3}$ (B) $\frac{5}{3}$ (C) 1 (D) $\frac{2}{3}$

41. If $\frac{5\pi}{2} < x < 3\pi$, then the value of the expression $\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}$ is: ▶
- (A) $-\cot \frac{x}{2}$ (B) $\cot \frac{x}{2}$ (C) $\tan \frac{x}{2}$ (D) $-\tan \frac{x}{2}$
42. Number of values of the parameter $\alpha \in [0, 2, \pi]$ for which the quadratic function, $(\sin \alpha)x^2 + 2\cos \alpha x + \frac{1}{2}(\cos \alpha + \sin \alpha)$ is the square of a linear function is:
- (A) 2 (B) 3 (C) 4 (D) 1
43. The value of $\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$ is equal to :
- (A) $\cot 3x$ (B) $\tan 3x$ (C) $3 \tan 3x$ (D) $\frac{3 - 9\tan^2 x}{3\tan x - \tan^3 x}$
44. If θ be an acute angle satisfying the equation $8\cos 2\theta + 8\sec 2\theta = 65$, then the value of $\cos \theta$ is equal to:
- (A) $\frac{1}{8}$ (B) $\frac{2}{3}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{3}{4}$
45. In which one of the following intervals the inequality, $\sin x < \cos x < \tan x < \cot x$ may hold good?
- (A) $\left(0, \frac{\pi}{4}\right)$ (B) $\left(\frac{3\pi}{4}, \pi\right)$ (C) $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$ (D) $\left(\frac{7\pi}{4}, 2\pi\right)$
46. Let $f(x) = a \sin x + c$, where a and c are real numbers and $a > 0$. Then $f(x) < 0 \forall x \in R$ if:
- (A) $c < -a$ (B) $c > -a$ (C) $-a < c < a$ (D) $c < a$
47. If $\tan x + \tan y = 25$ and $\cot x + \cot y = 30$, then the value of $\tan(x+y)$ is:
- (A) 150 (B) 200 (C) 250 (D) 100
48. In a right-angled triangle, the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are:
- (A) $\frac{\pi}{3}$ and $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$ and $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ and $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ and $\frac{3\pi}{10}$
49. If m and n are positive integers satisfying $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta + \cos 10\theta = \frac{\cos m\theta \cdot \sin n\theta}{\sin \theta}$ then $(m+n)$ is equal to:
- (A) 9 (B) 10 (C) 11 (D) 12
50. Consider the polynomial $P(x) = (x - \cos 36^\circ)(x - \cos 84^\circ)(x - \cos 156^\circ)$. The coefficient of x^2 is:
- (A) 0 (B) 1 (C) $-\frac{1}{2}$ (D) $\frac{\sqrt{5}-1}{2}$
51. Suppose that ' a ' is a non-zero-real number for which $\sin x + \sin y = a$ and $\cos x + \cos y = 2a$. The value of $\cos(x-y)$, is:
- (A) $\frac{3a^2-2}{2}$ (B) $\frac{7a^2-2}{2}$ (C) $\frac{9a^2-2}{2}$ (D) $\frac{5a^2-2}{2}$

- 52.** Let $P(x) = \sqrt{(\cos x + \cos 2x + \cos 3x)^2 + (\sin x + \sin 2x + \sin 3x)^2}$, then $P(x)$ may be equal to:
(A) $1 + 2\cos x$ **(B)** $1 + \sin 2x$ **(C)** $1 - 2\cos x$ **(D)** None of these
- 53.** If $\frac{\cos \theta}{a} = \frac{\sin \theta}{b}$, then $\frac{a}{\sec 2\theta} + \frac{b}{\operatorname{cosec} 2\theta}$ is equal to:
(A) a **(B)** b **(C)** $\frac{a}{b}$ **(D)** $a + b$
- 54.** $f(\theta) = |\sin \theta| + |\cos \theta|$, $\theta \in R$, then:
(A) $f(\theta) \in [0, 2]$ **(B)** $f(\theta) \in [0, \sqrt{2}]$ **(C)** $f(\theta) \in [0, 1]$ **(D)** $f(\theta) \in [1, \sqrt{2}]$
- 55.** If $0 \leq x \leq \frac{\pi}{3}$ then range of $f(x) = \sec\left(\frac{\pi}{6} - x\right) + \sec\left(\frac{\pi}{6} + x\right)$ is:
(A) $\left(\frac{4}{\sqrt{3}}, \infty\right)$ **(B)** $\left[\frac{4}{\sqrt{3}}, \infty\right)$ **(C)** $\left(0, \frac{4}{\sqrt{3}}\right]$ **(D)** $\left(0, \frac{4}{\sqrt{3}}\right)$